Generalized Symmetry in QFTs & Applications to QCD

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Solving Quantum Field Theories

UV ← microscopic description
- Quantum Chromodynamics
- (Lattice) quantum many-body systems
- Conformal field theories (CFT_{UV})

Renormalization Group (RG) flow

IR ← macroscopic description
- Nonlinear σ-model (chiral Lagrangian)
- Fermi liquid, spin liquid (TQFT), ...
- CFT_{IR}.

This is a very tough problem!

Power of Symmetry

Sometimes, we can know about Low-energy dynamics using Symmetry without solving microscopic Hamiltonian.

e.g. In '60s, people didn't know about Quantum Chromodynamics (QCD).
fundamental theory of strong interaction.

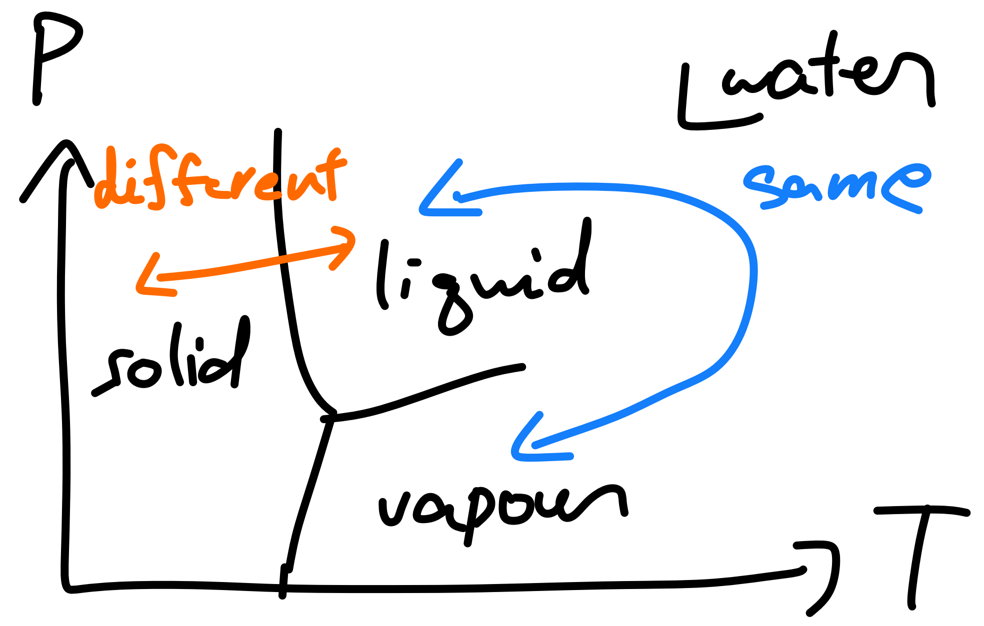
current algebra (chiral effective Lagrangian)
$\Longrightarrow$ successful description of low-energy properties of strong interaction

Why this was possible?

$\boxed{\text{Universality due to SSB of chiral symmetry.}}$

(Some of) Important Theorems related to Symmetry

- **Landau's criterion of phases of matter**



  If symmetry breaking patterns are different, there have to be a phase transition separating those states.

- **Nambu-Goldstone theorem**

  If continuous symmetry is spontaneously broken, there are massless NG bosons.
  They have derivative couplings, so they interact weakly at low-energies.

- **'t Hooft anomaly matching**

  Quantum anomaly associated with gauging of global symmetry is RG invariant.

Continuous Symmetry in QFT

Noether : If classical action $S[\phi]$ is invariant under continuous transformation, then there is a conserved current J^μ :

$$\partial_\mu J^\mu = 0.$$

$$\Downarrow$$

In QFT, this becomes Ward–Takahashi identity :

$$\langle \partial_\mu J^\mu(x) \; O_1(x_1) \cdots O_n(x_n) \rangle = \sum_i \delta(x - x_i) \langle O_1(x_1) \cdots \delta O_i(x_i) \cdots O_n(x_n) \rangle .$$

$$\Downarrow$$

Various theorems related to symmetry.

Generalization of Symmetry

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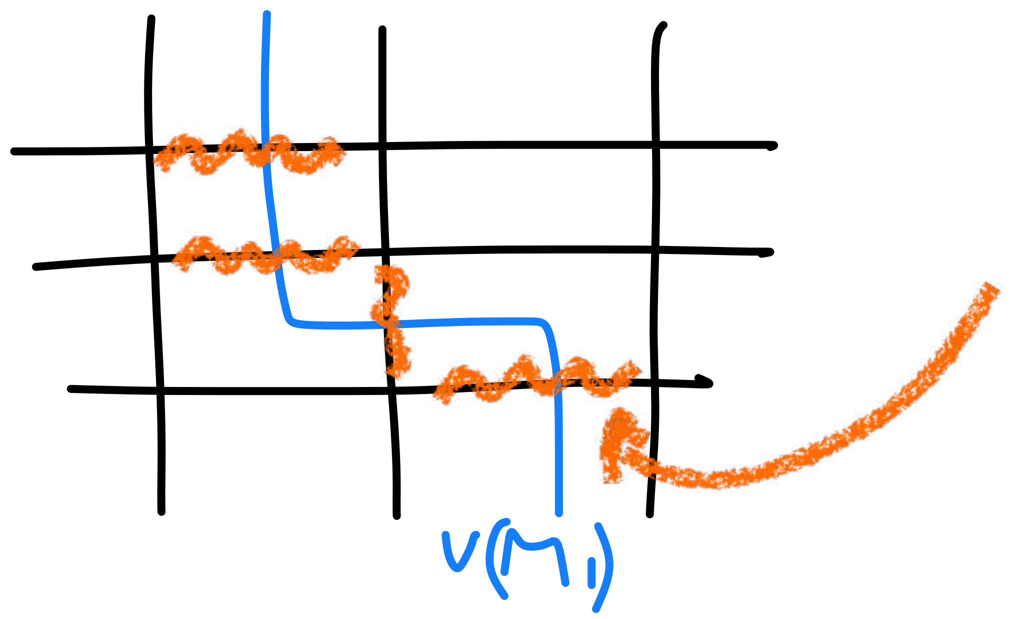
Generalization of Ward-Takahashi identity

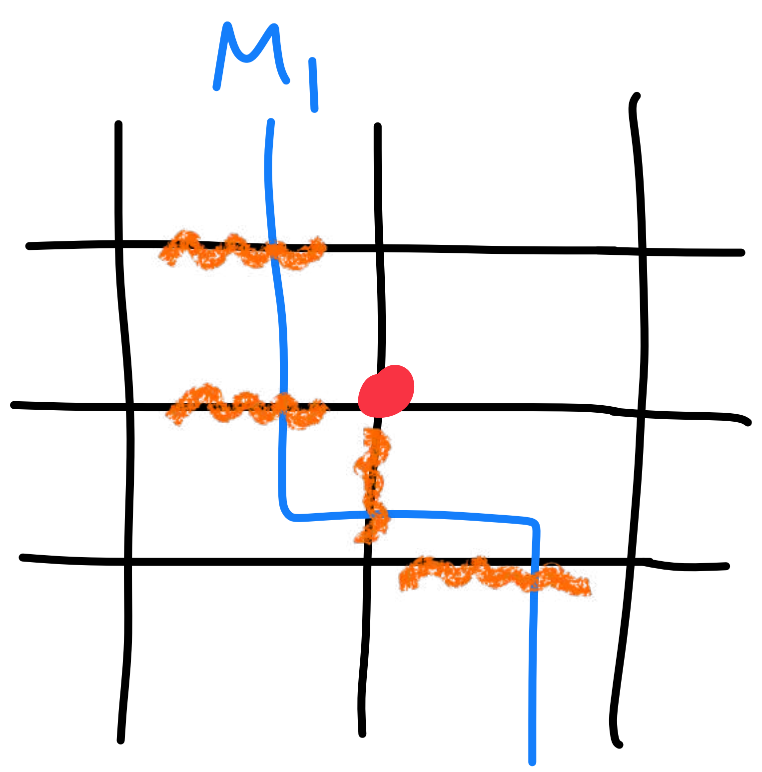
Example WT-type identity for $\mathbb{Z}_2$ symmetry of Ising model

$$H_{\text{Ising}} = J \sum_{\substack{\langle x,x'\rangle:\text{nearest}\\ \text{neighbors}}} s(x)\,s(x') \qquad (s(x)=\pm 1)$$

$V(M_1)$: defect operator



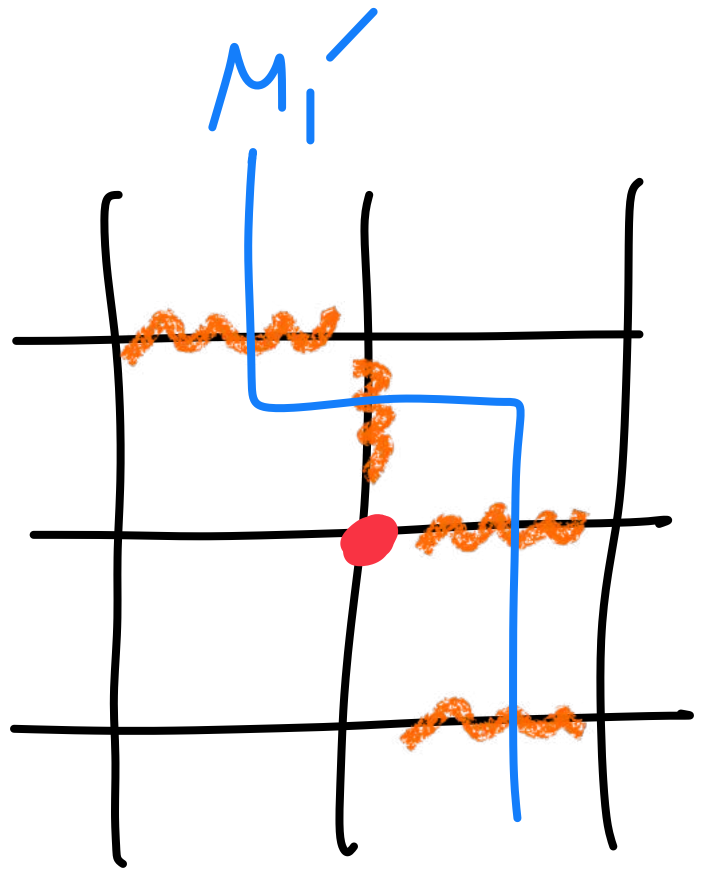
For these $\langle x,x'\rangle$, change $J \to -J$ in the Hamiltonian.



$\mathbb{Z}_2$ operation for $S(x)$ at ● :

$s \to -s$

$\Longrightarrow$



WT-type identity

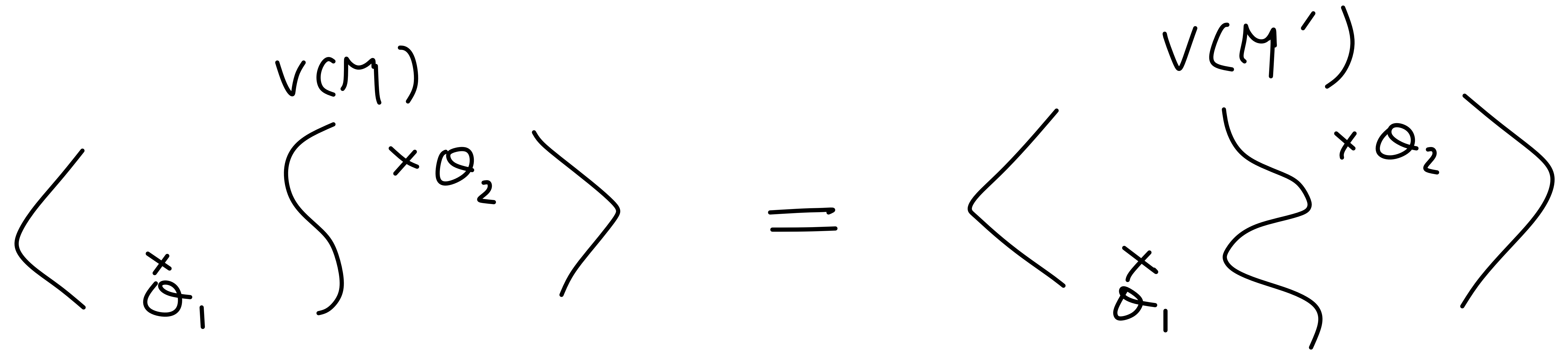
$$\langle V(M_1) \cdots \rangle = \langle V(M_1') \cdots \rangle.$$

Modern Definition of (Generalized) Symmetry

Take-home message

$$\boxed{\text{Symmetry} = \text{Topological defect operators}}$$

Topological



$\Longleftrightarrow$ Conservation Law

(Continuous sym : $V(M) = e^{i\alpha Q(M)} = e^{i\alpha \int_M *(J_\mu dx^\mu)}$.

$\mathbb{Z}_2$ of Ising : $V(M)$ = J in H is replaced by $-J$ when $\langle x, x' \rangle$ crosses M.)

Ordinary Symmetry in Modern Viewpoints

d-dim. QFT has a global symmetry G.

$\overset{\text{def}}{\Longleftrightarrow}$

- $\exists\, V_g(M_{d-1})$: topological codim-1 defect operator for each $g \in G$

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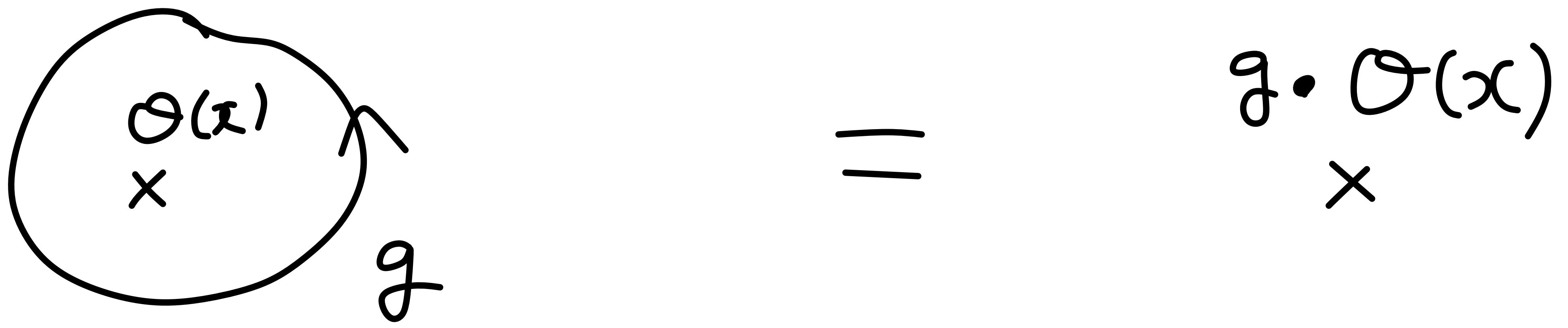
- 

(Valid for both continuous and discrete symmetries)

Various generalizations

- **p-form symmetry** (Gaiotto, Kapustin, Seiberg, Willet '14)

  Topological defects have codim-$(p+1)$: $V_g(M_{d-p-1})$.

  (* Esp., 1-form symmetry generalizes the center sym. in gauge theories.)

- **n-group symmetry** (Sharpe '15, Cordova, Dumitrescu, Intriligator '17, YT, Ünsal '19 ...)

  $\approx$ Mixture of 0-, 1-, ..., (n-1)-form symmetries.

- **non-invertible symmetry** (Bhardwaj, Tachikawa '17, ... in 2d QFTs. Nguyen, YT, Ünsal '21, Koide, Nagoya, Yamaguchi '21, ... in 3d, 4d)

  Transformation rule does not form a group

Application 1: Phase diagram of Fradkin-Shenker's model

Fradkin - Shenker's (non-) complementarity.

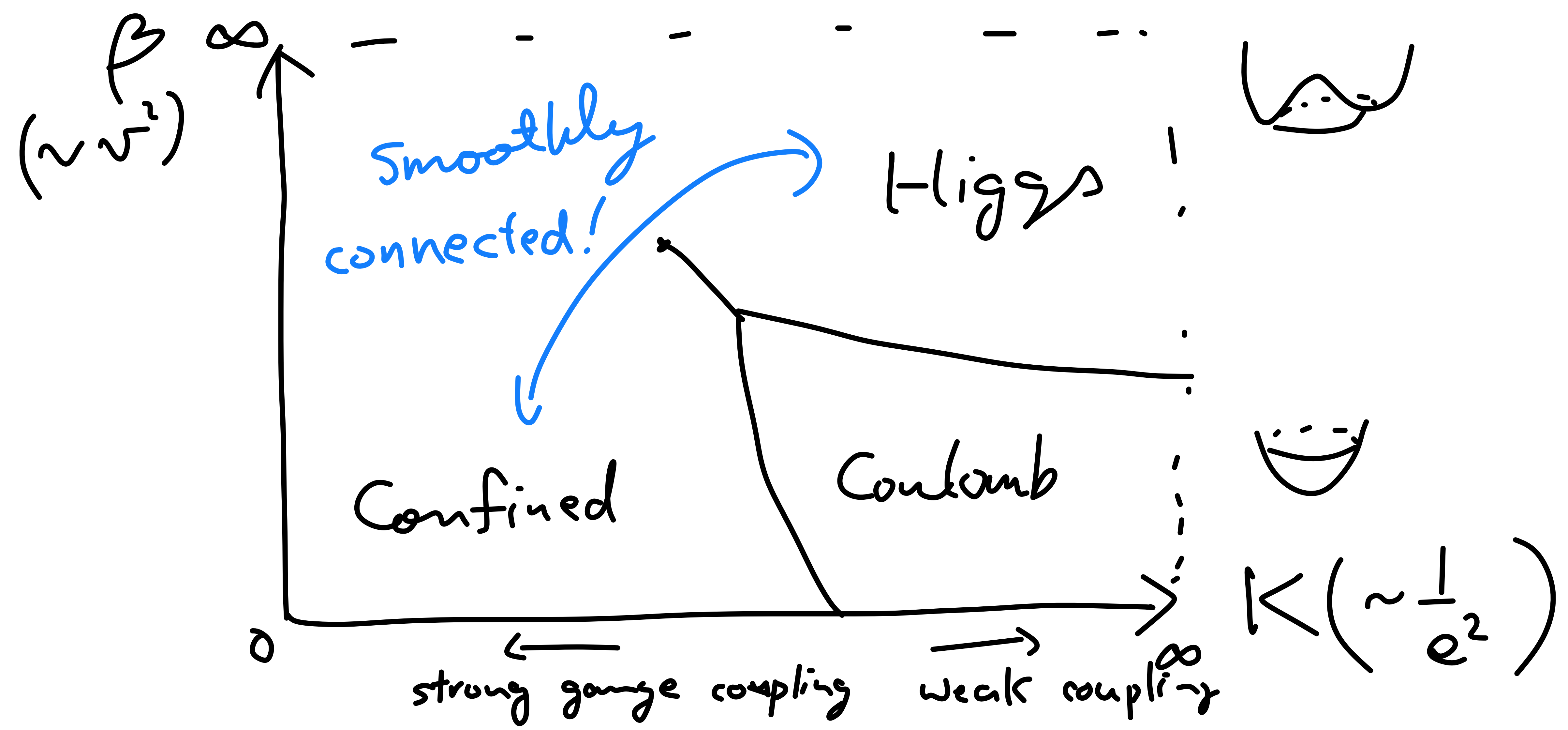
Consider the compact $U(1)$ gauge theory coupled to charge-q scalar ($q=1,2,3,\cdots$)

$$S = \frac{1}{2e^2}\int da \wedge * da + \int\left\{ \left|(\partial_\mu + i q a_\mu)\phi\right|^2 + g(|\phi|^2 - v^2)^2 \right\}.$$

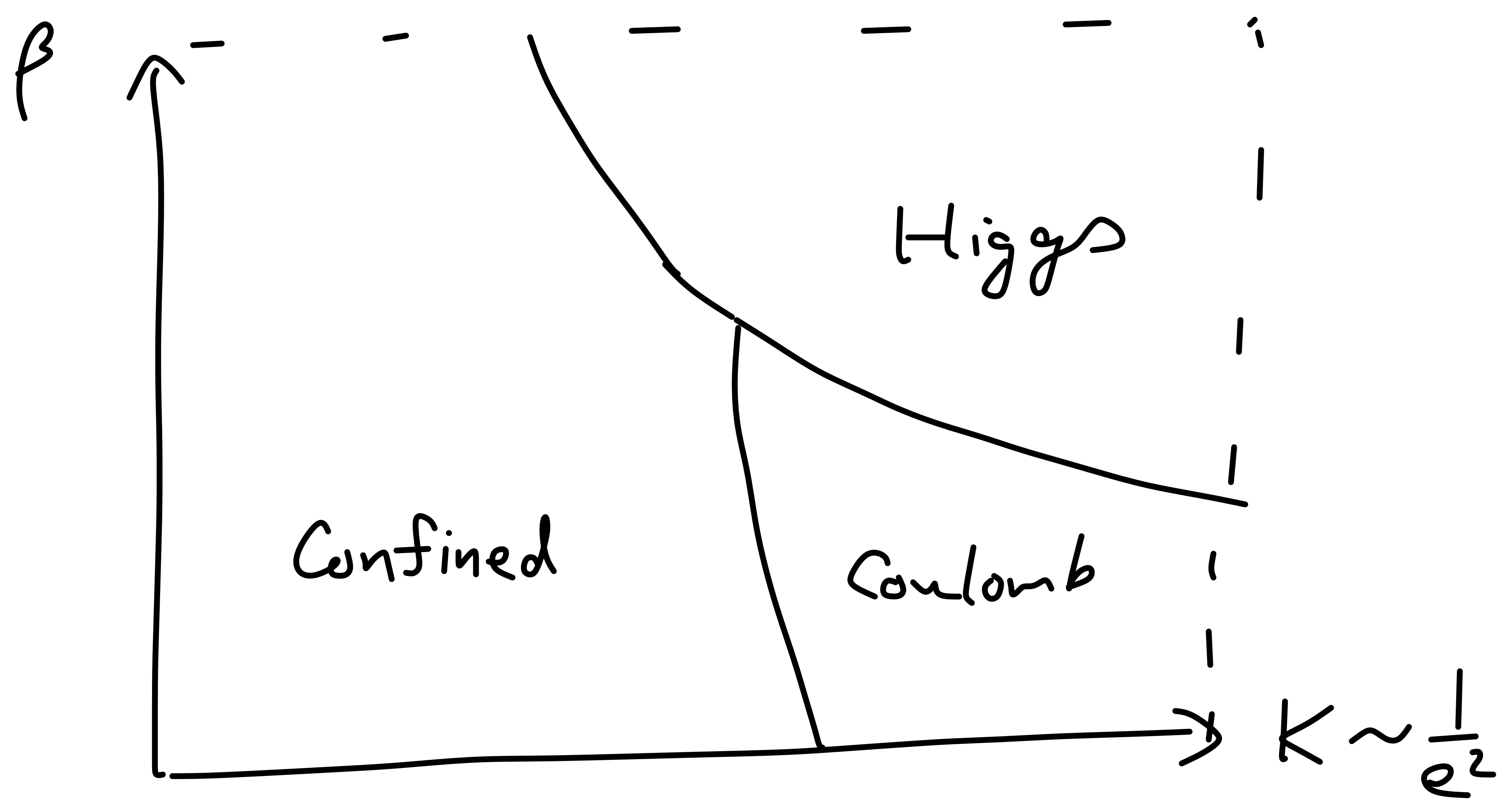
(In Fradkin, Shenker's paper ('79), the lattice version is considered, ($\phi \leftrightarrow e^{i\theta}$)

$$S = \beta \sum_i \cos(\partial_\mu \theta + q a_\mu) + K \sum_{\square} \cos(f_{\mu\nu}).$$
)

$\underline{q=1}$ (Confined & Higgs phases are the same)



$\underline{q \geq 2}$ (They are different)



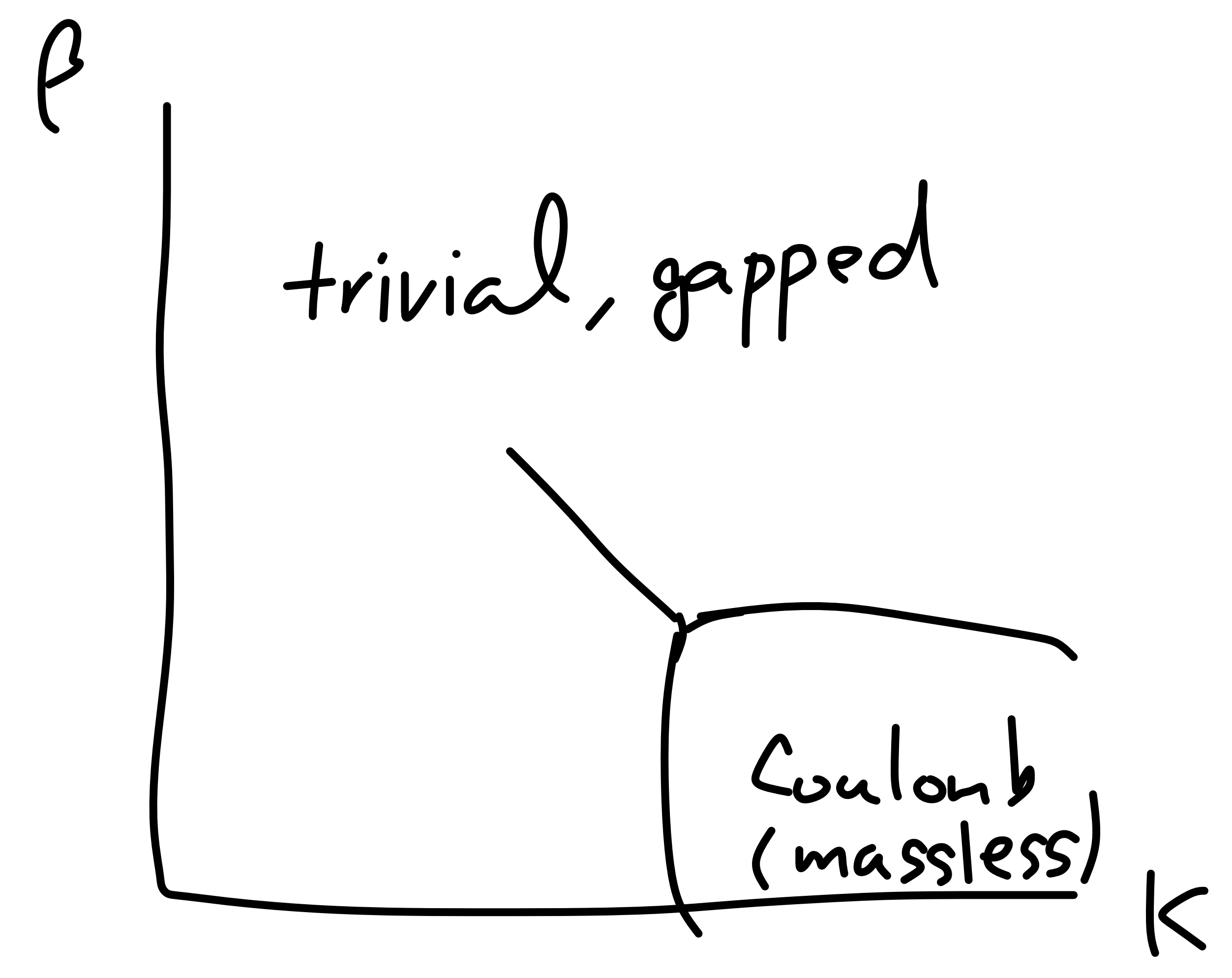
Fradkin - Shenker revisited (Application of 1-form symmetry)

They consider charge-q $U(1)$-Higgs model on a lattice

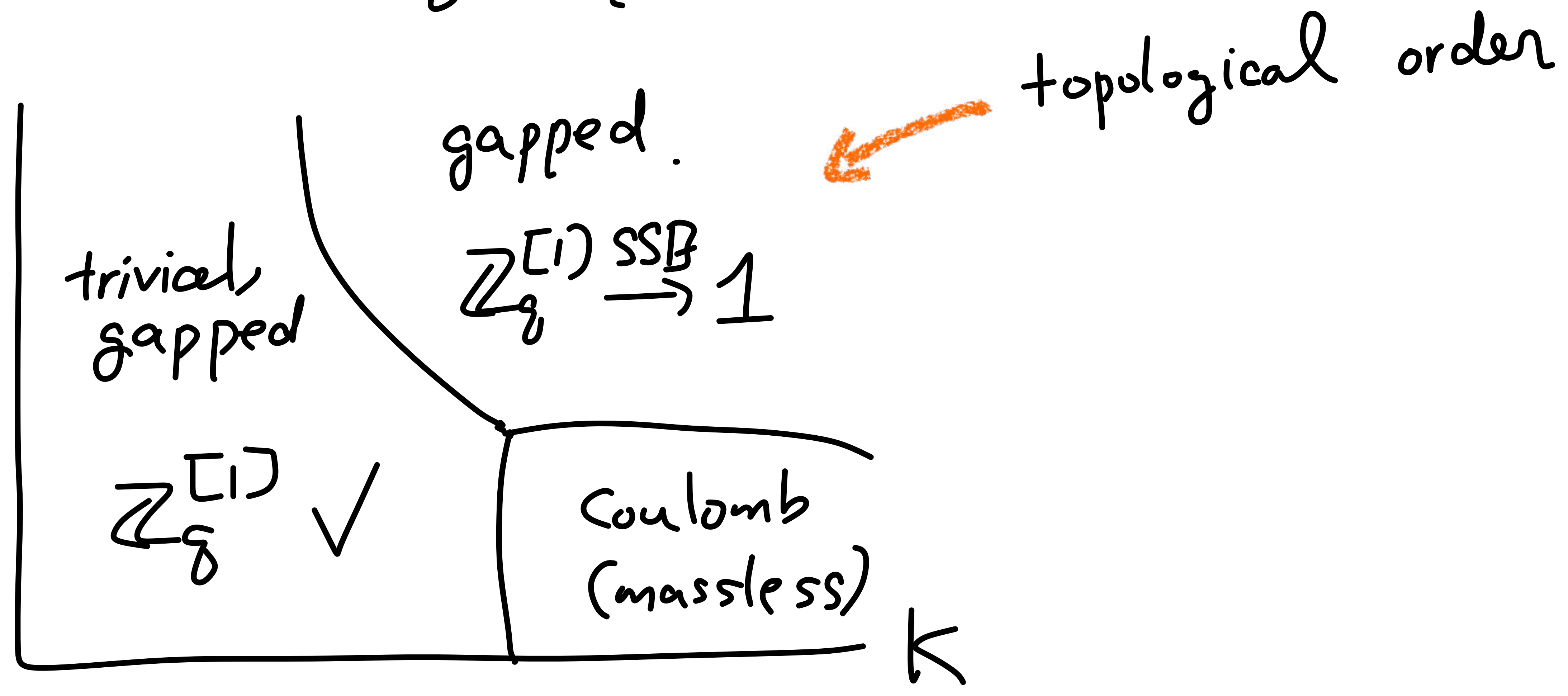
$$S = \beta \sum_{i,\mu} \cos(\partial_\mu \theta + q a_\mu) + K \sum_{\square} \cos(f_{\mu\nu})$$

$$\left(\longleftrightarrow \quad S = \frac{1}{2e^2}\int |da|^2 + \underbrace{\int \left\{ |(\partial_\mu + i q a_\mu)\phi|^2 + \cdots \right\}}_{U(1)_E^{[1]} \xrightarrow{\text{explicit}} \mathbb{Z}_q^{[1]}} + \underbrace{\text{monopoles}}_{U(1)_M^{[1]} \xrightarrow{\text{explicit}} \times} \right).$$

$q = 1$ (No symmetry)



$q \geq 2$ ($\mathbb{Z}_q^{[1]}$ symmetry)



Application 2 : Refined understanding on duality relations

Montonen-Olive duality for $\mathcal{N}=4$ super Yang-Mills

We know that Maxwell eq.

$$\begin{cases} \nabla \cdot \mathbb{E} = \rho_e \\ \nabla \cdot \mathbb{B} = \rho_m \end{cases} \qquad \begin{cases} \nabla \times \mathbb{E} + \dot{\mathbb{B}} = \mathbb{j}_m \\ \nabla \times \mathbb{B} - \dot{\mathbb{E}} = \mathbb{j}_e \end{cases}$$

has the electromagnetic duality

$$\begin{pmatrix} \mathbb{E} \\ \mathbb{B} \end{pmatrix} \to \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \mathbb{E} \\ \mathbb{B} \end{pmatrix}.$$

In $\mathcal{N}=4$ SYM, this is believed to be true quantum mechanically:

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}. \qquad \text{(complex gauge coupling)}$$

$$S: \tau \to -\frac{1}{\tau}, \qquad T: \tau \to \tau + 1 \quad \text{form } SL(2,\mathbb{Z}) \text{ duality.}$$

It was known by Montonen & Olive ('77) that

$$S: \ SU(2) \text{ gauge theory} \longleftrightarrow SO(3) \text{ gauge theory}.$$

In modern understanding,

$$4d \ SO(3)\left(= \frac{SU(2)}{\mathbb{Z}_2}\right) \text{ gauge theory} \xleftarrow{\mathbb{Z}_N^{[1]} \text{ gauging}} 4d \ SU(2) \text{ gauge theory}.$$

= introduction of flat $\mathbb{Z}_N$ 2-form gauge field b.

Here, depending on the choice of $\boxed{\text{counterterm} \quad i\frac{Nk}{4\pi}\int b\wedge b}$,

we have two different $SO(3)$ theories : $SO(3)_+$, $SO(3)_-$.

(Aharony, Seiberg, Tachikawa '13). This was missed in the past.

$$T \circlearrowright SU(2) \xleftrightarrow{\ S\ } SO(3)_+ \xleftrightarrow{\ T\ } SO(3)_- \circlearrowleft S.$$

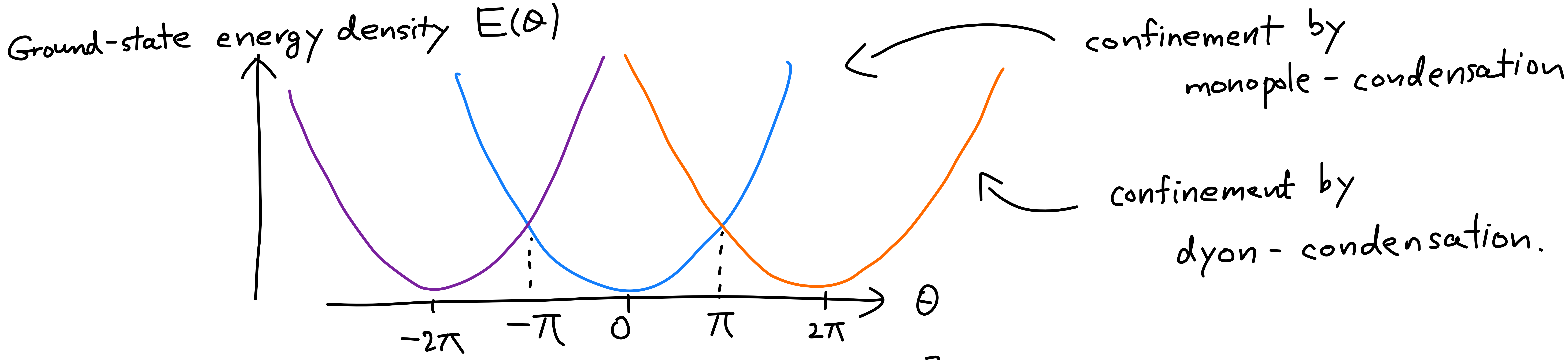
Application 3 : θ-dependence of 4d gauge theories

YM theory at finite θ.

4d gauge theory has two renormalizable terms:

$$\underbrace{\frac{1}{g^2}\int F_{\mu\nu}F_{\mu\nu}}_{\text{kinetic term}} + i\theta\int \underbrace{\varepsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}}_{\text{topological term }(=\text{ instanton number})}.$$

When confinement occurs at any θ, the conjectured phase diagram is



Q. All confinement phases have unbroken $\mathbb{Z}_N^{[1]}$. Is the phase transition at $\theta=\pi$ accidental?

A. These confinement states are different as Symmetry-Protected Topological (SPT) states,

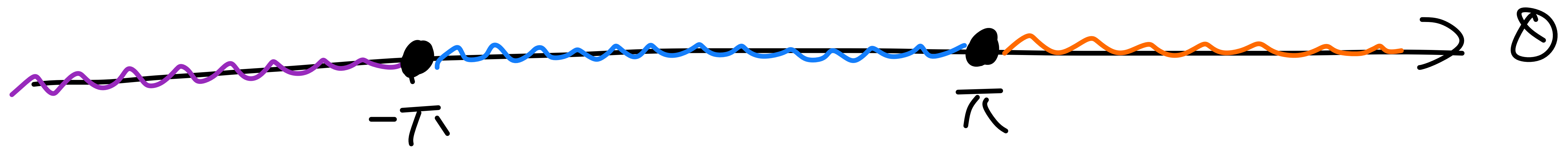
$\Rightarrow$ Phase transition is mandatory. (Gaiotto, Kapustin, Komargodski, Seiberg '17)

B : $\mathbb{Z}_N$ 2-form gauge field (= Background gauge field for $\mathbb{Z}_N^{[1]}$)

$$Z_{\theta+2\pi}[B] = e^{i\frac{N}{4\pi}\int B\wedge B} \times Z_\theta[B].$$

$\uparrow$ 2π-periodicity of θ is violated by a local counterterm of B.

$Z_\theta[B] \sim e^{-i\frac{N}{4\pi}\int B\wedge B}$ $\quad$ $Z_\theta[B] \sim 1$ $\quad$ $Z_\theta[B] \sim e^{i\frac{N}{4\pi}\int B\wedge B}$



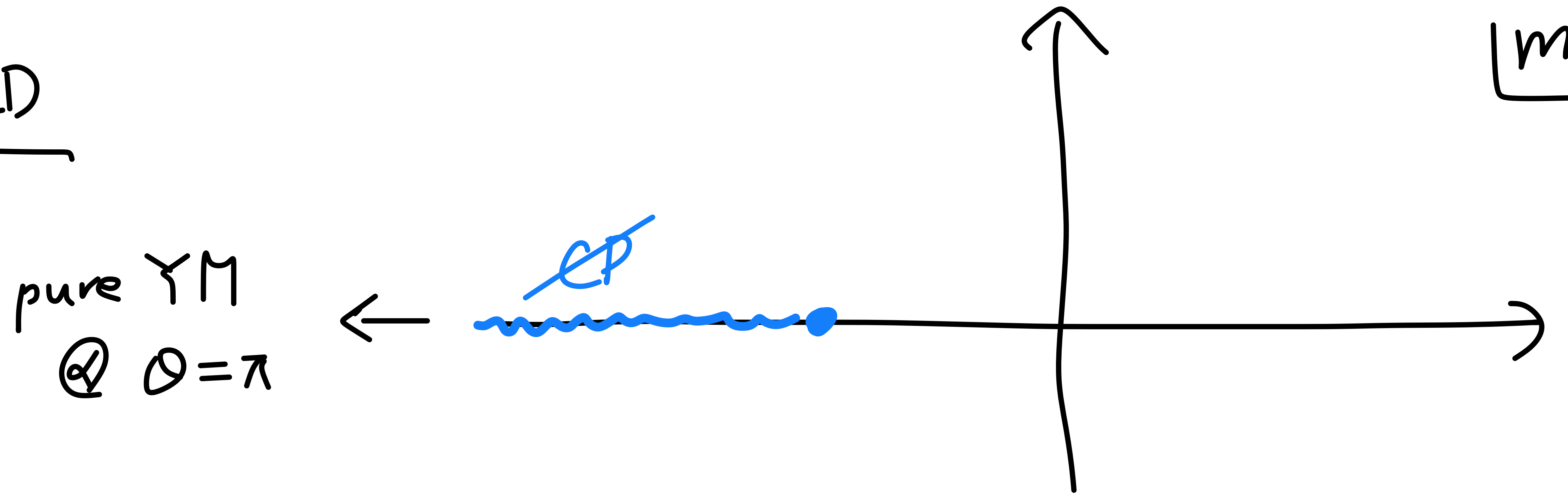
Without 1-form symmetry

Adding fundamental quarks (i.e. pure YM $\Rightarrow$ QCD),

$\mathbb{Z}_N^{[1]}$ is gone.

<u>1-flavor QCD</u>

$m e^{i\theta}$ ($m \geq 0$: quark mass)



In this case, distinction between monopole- & dyon-induced confinement disappears.

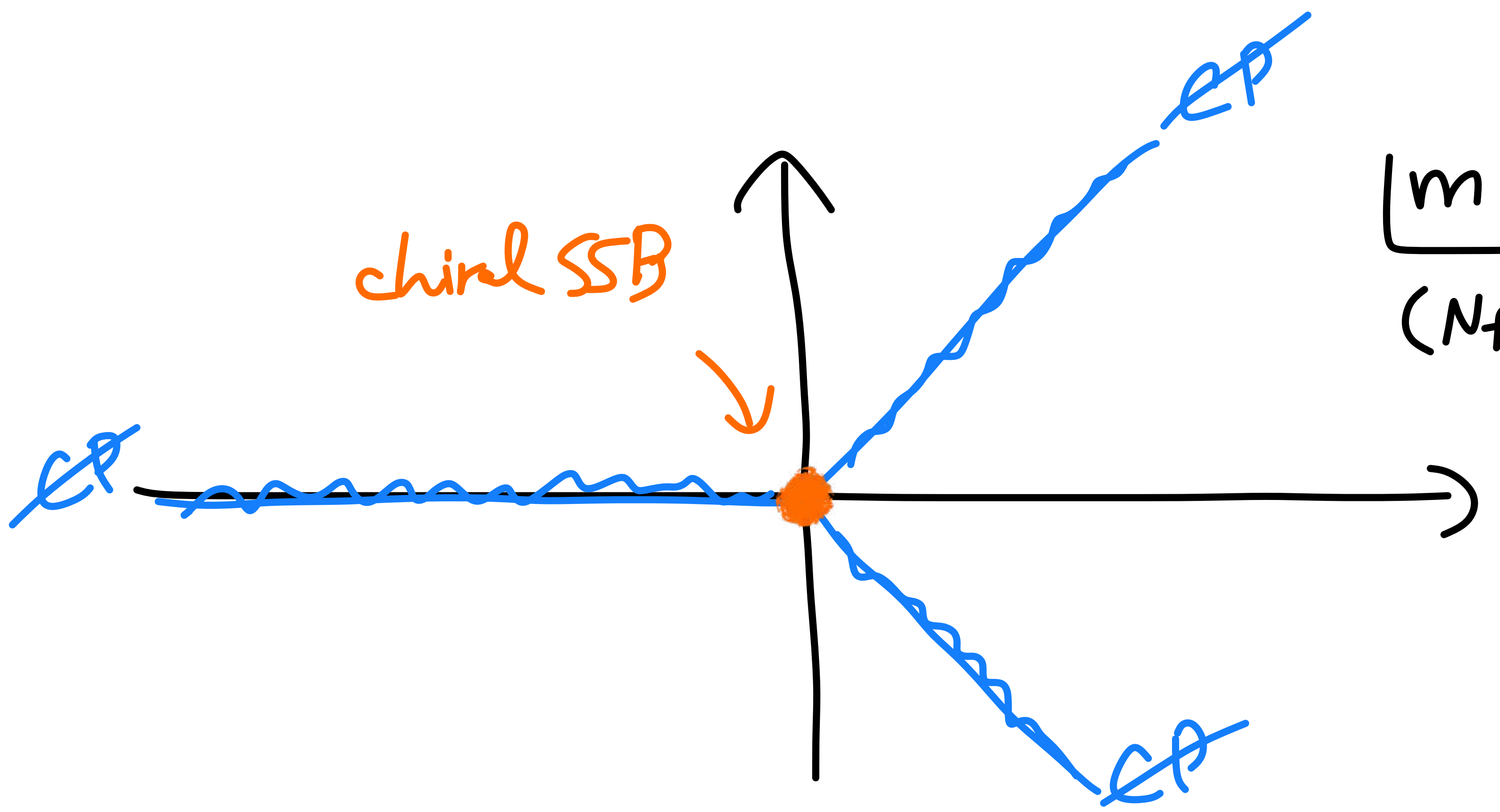
<u>N_f-flavor QCD ($N_f \geq 2$)</u>

When $\gcd(N_c, N_f) > 1$,

we have a remnant of anomaly.

(YT, Kikuchi '17, Shimizu, Yonekura '17, Gaiotto, Komargodski, Seiberg '17)

$m e^{i\theta/N_f}$ ($N_f = 3$) ($m \geq 0$: flavor-symmetric quark mass)



Application 4 : Adiabatic continuity

Large-N volume independence.

Eguchi, Kawai ('82) showed that

4d lattice gauge theory on T^4

$=$ 1 plaquette model

as long as $\mathbb{Z}_N^{[1]}$ is unbroken.

To satisfy this requirement, twisted EK model was proposed,

where nontrivial 't Hooft twisted b.c. is chosen (Gonzalez-Arroyo, Okawa '83).

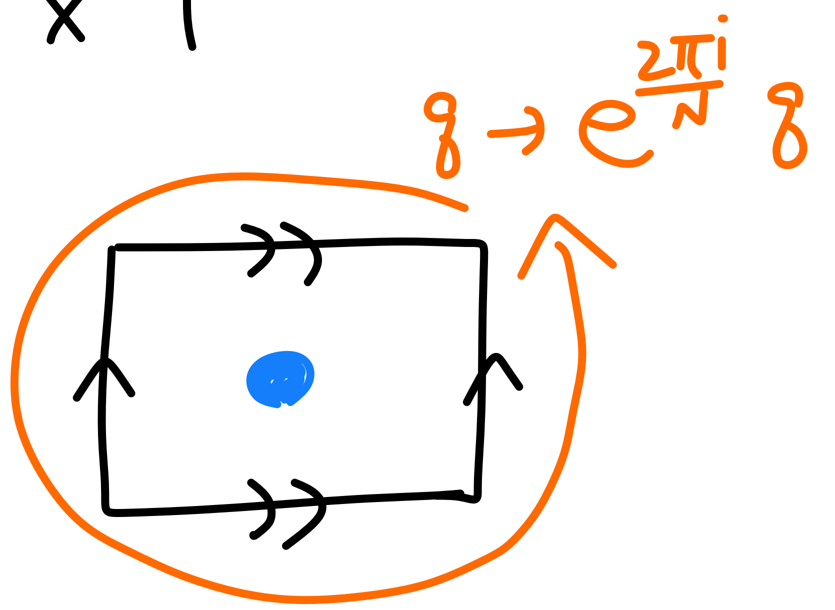
$=$ Background gauge field for $\mathbb{Z}_N^{[1]}$

Semiclassical description of confinement.

(YT, Ünsal '22)

Conjecture

YM, QCD on $\mathbb{R}^4$ $\xleftrightarrow{\text{Adiabatic Continuity}}$ YM, QCD on $\mathbb{R}^2 \times T^2$ w/ 't Hooft flux



Results

For small $\mathbb{R}^2 \times T^2$ w/ 't Hooft flux, dilute gas of Center Vortex predicts

- (YM theory) $\quad E_k(\theta) \sim -\Lambda^2 (\Lambda L)^{5/3} \cos\left(\frac{\theta - 2\pi k}{N}\right)$. (Multi-branch vacua)
- ($\mathcal{N}=1$ SYM) $\quad \langle \mathrm{tr}(\lambda\lambda) \rangle \sim \Lambda^3 e^{i(\theta - 2\pi k)/N}$ (Discrete chiral SSB)
- (QCD w/ non-commuting flavor twist ($N_f = N_c$)) $\quad \langle \mathrm{tr}_{cf}(\bar{\psi}_L)\, \mathrm{tr}_{cf}(\psi_R) \rangle \sim \Lambda^3 e^{i(\theta - 2\pi k)/N}$.
- (QCD w/ $U(1)_B$ monopole flux) $\quad S_{\mathrm{eff}} \sim \int \left\{ |dU|^2 + \frac{1}{12\pi} \mathrm{tr}(U^\dagger dU)^3 + \chi_{\mathrm{top}} \left(i \ln \det U - \theta\right)^2 \right\}$ (Witten-Veneziano formula)

Summary

Take-home message

$$\boxed{\text{Symmetry} = \text{Topological defect operators}}$$

$\Rightarrow$ New aspects of strongly-coupled QFTs.